EXAM 3 REVIEW

Note Title 11/7/2012

1. $f(x) = \frac{7}{x-3}$ $g(x) = 2x+11$

$(f \circ g)(x) = f(2x+11) = \frac{7}{2x+11-3} = \boxed{\frac{7}{2x+8}}$

Domain: $2x+8 \neq 0$

$2x \neq -8$

$\boxed{\{x \mid x \neq -4\}}$

2. $f(x) = 2x+5$ $g(x) = \sqrt{x-3}$

$(g \circ f)(x) = g(2x+5) = \sqrt{2x+5-3} = \boxed{\sqrt{2x+2}}$

Domain: $2x+2 > 0$

$2x > -2$

$\boxed{\{x \mid x > -1\}}$

3. $f(x) = \frac{2}{x+4}$ $g(x) = \frac{3}{5x}$

$(f \circ g)(x) = f\left(\frac{3}{5x}\right) = \frac{2}{\frac{3}{5x}+4} = \left(\frac{2}{\frac{3}{5x}+4}\right)\left(\frac{5x}{5x}\right) = \boxed{\frac{10x}{3+20x}}$

$x \neq 0$

$3+20x \neq 0$

$20x \neq -3$ $x \neq \frac{-3}{20}$

$\boxed{\text{Domain: } \{x \mid x \neq 0, -\tfrac{3}{20}\}}$

4. $f(x) = \sqrt{x+2}$ $g(x) = 2x^2+1$

$(g \circ f)(7)$

$f(7) = \sqrt{7+2} = \sqrt{9} = 3$

$g(3) = 2(3)^2+1 = \boxed{19}$

5. A function is one-to-one if each x has only one y and each y has only one x. If a graph passes the horizontal line test, the function is one-to-one.

To determine whether two functions are inverses, check whether $(f\circ g)(x)=x$ and $(g\circ f)(x)=x$.

6. $f(x)=3x-6 \quad g(x)=\frac{1}{3}x+2$

$(f\circ g)(x)=3\left(\frac{1}{3}x+2\right)-6 = x+6-6=x$

$(g\circ f)(x)=\frac{1}{3}(3x-6)+2 = x-2+2=x$

$\boxed{\text{inverses}}$

7. $f(x)=4x+5 \quad g(x)=\frac{1}{4}x-5$

$(f\circ g)(x) = 4\left(\frac{1}{4}x-5\right)+5 = x-20+5 = x-15$

$(g\circ f)(x) = \frac{1}{4}(4x+5)-5 = x+\frac{5}{4}-5 = x-\frac{15}{4}$

$\boxed{\text{not inverses}}$

To find inverses, swap x & y and solve for y.
Remember: Domain of f = Range of f^{-1}
Range of f = Domain of f^{-1}

8. $f(x)=\frac{4}{x+3}$

$x=\frac{4}{y+3}$

$x(y+3)=4$

$xy+3x=4$

$xy=4-3x$

$y=\frac{4-3x}{x}$

$\boxed{f^{-1}(x)=\frac{4-3x}{x}}$

Domain of f: $\{x \mid x\neq -3\}$
Domain of f^{-1}: $\{x \mid x\neq 0\}$
Range of f: $\{y \mid y\neq 0\}$
Range of f^{-1}: $\{y \mid y\neq -3\}$

9. $f(x)=\frac{2x+3}{5x-7}$

$x=\frac{2y+3}{5y-7}$

$x(5y-7)=2y+3$

$5xy-7x=2y+3$

$5xy-2y=7x+3$

$y(5x-2)=7x+3$

$y=\frac{7x+3}{5x-2}$

$\boxed{f^{-1}(x)=\frac{7x+3}{5x-2}}$

Domain of f: $\{x \mid x\neq \frac{7}{5}\}$
Domain of f^{-1}: $\{x \mid x\neq \frac{2}{5}\}$
Range of f: $\{y \mid y\neq \frac{2}{5}\}$
Range of f^{-1}: $\{y \mid y\neq \frac{7}{5}\}$

10. $f(x) = \log_2(x-3)$

$x = \log_2(y-3)$

$2^x = y-3$

$2^x + 3 = y$

$\boxed{f^{-1}(x) = 2^x + 3}$

Domain of f: $x-3>0$

$\{x \mid x>3\}$

Range of f: $\mathbb{R}$

Domain of f^{-1}: $\mathbb{R}$

Range of f^{-1}: $\{y \mid y>3\}$

11. $f(x) = 2e^{x+5}$

$x = 2e^{y+5}$

$\frac{x}{2} = e^{y+5}$

$\ln\left(\frac{x}{2}\right) = y+5$

$y = \ln\left(\frac{x}{2}\right) - 5$

$\boxed{f^{-1}(x) = \ln\left(\frac{x}{2}\right) - 5}$

Domain of f^{-1}: $\frac{x}{2} > 0$

$\{x \mid x>0\}$

Range of f^{-1}: $\mathbb{R}$

Domain of f: $\mathbb{R}$

Range of f: $\{y \mid y>0\}$

12. $f(x) = x^2 - 3, \quad x \geq 0$

$x = y^2 - 3, \quad y \geq 0$

$x + 3 = y^2$

$y = \sqrt{x+3}$ ← since $y \geq 0$, this must be the positive root.

$\boxed{f^{-1}(x) = \sqrt{x+3}}$

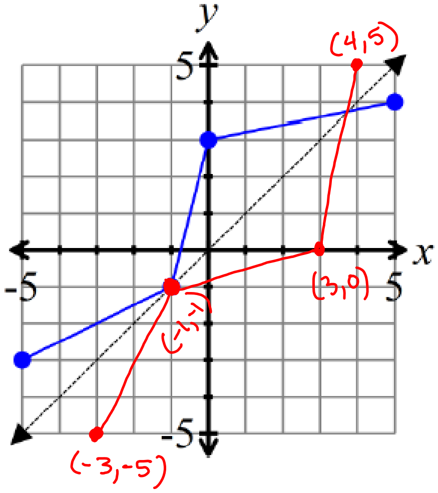
Domain of f: $\{x \mid x \geq 0\}$

Range of f: $\{y \mid y \geq -3\}$

Domain of f^{-1}: $\{x \mid x \geq -3\}$

Range of f^{-1}: $\{y \mid y \geq 0\}$

13.



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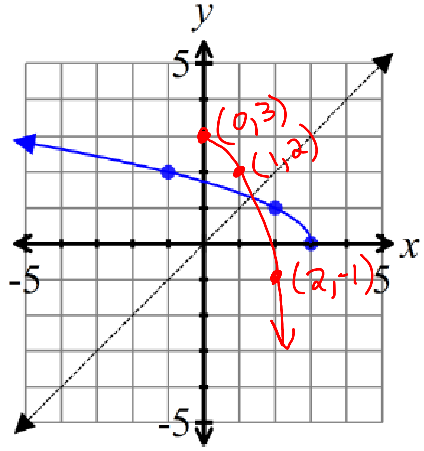

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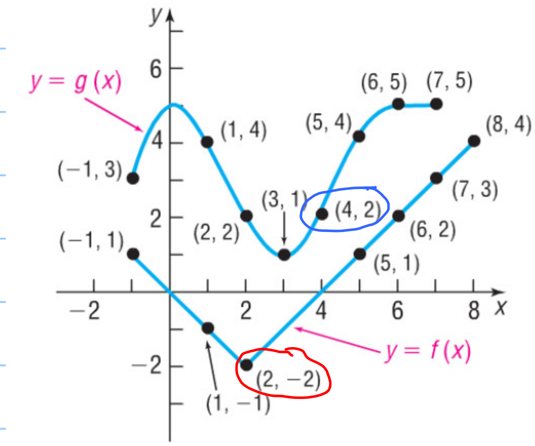


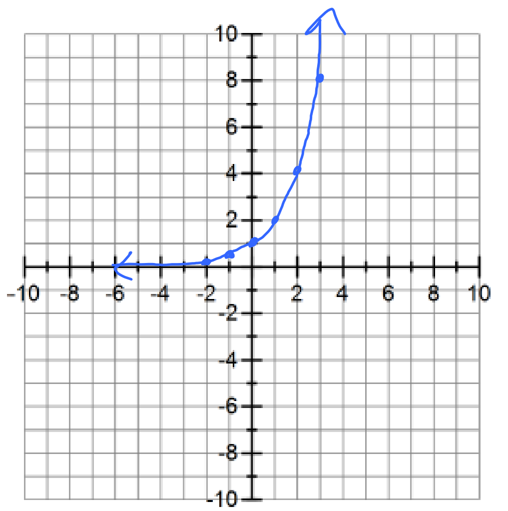
$(f \circ g)(4)$

$g(4) = 2$

$f(2) = \boxed{-2}$

16. $f(x) = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4



Domain: $\mathbb{R}$

Range: $(0, \infty)$

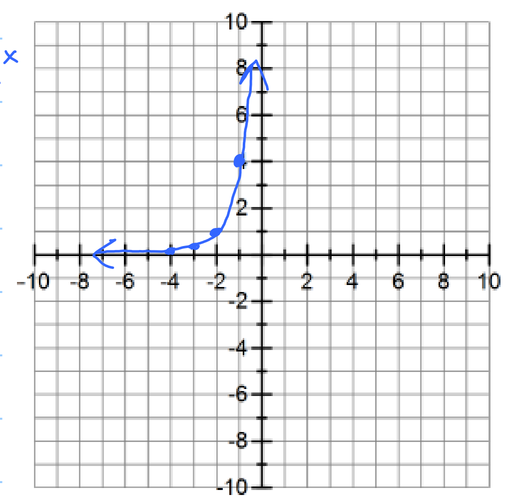
Horiz. Asymp: $y = 0$

17. $f(x) = 4^{x+2}$

Parent: $y = 4^x$

$\leftarrow 2$

x	y
-4	1/16
-3	1/4
-2	1
-1	4
0	16



Domain: $\mathbb{R}$

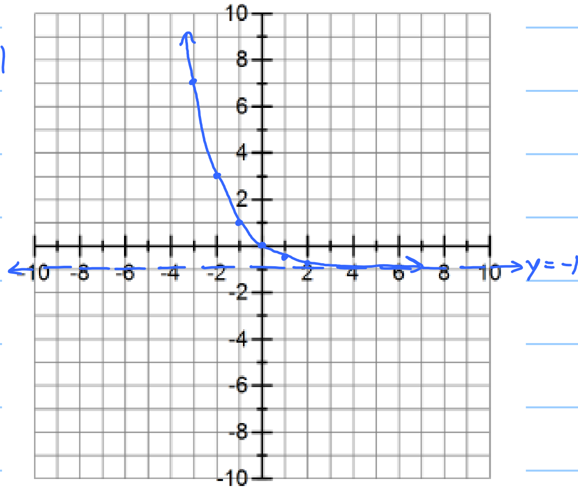
Range: $(0, \infty)$

Horiz. Asymp: $y = 0$

18. $f(x)=\left(\frac{1}{2}\right)^x - 1$

parent: $y=\left(\frac{1}{2}\right)^x$ ↓1

x	y
-2	3
-1	1
0	0
1	$-\frac{1}{2}$
2	$-\frac{3}{4}$



Domain: $\mathbb{R}$

Range: $(-1,\infty)$

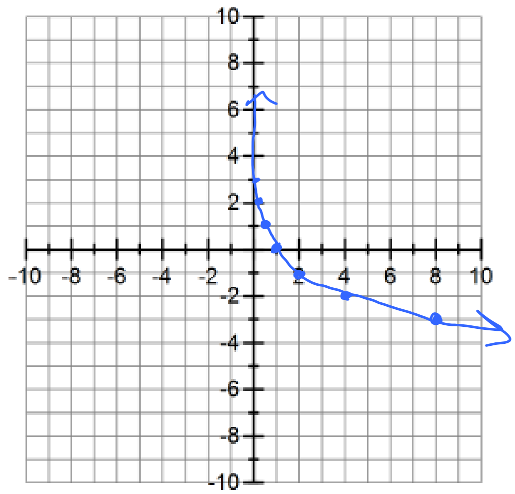
Horiz. Asymp: $y=-1$

19. $f(x)=\log_{1/2} x$

$y=\log_{1/2} x$

$x=\left(\frac{1}{2}\right)^y$

x	y
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2



Domain: $x>0$

$\{x \mid x>0\}$

Range: $\mathbb{R}$

Vert. Asymp: $x=0$

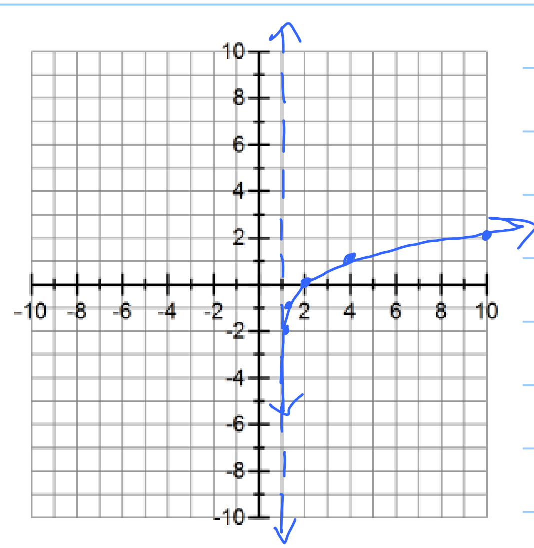
20. $f(x)=\log_3(x-1)$

$y=\log_3(x-1)$

$3^y=x-1$

$x=3^y+1$

x	y
$1\frac{1}{9}$	-2
$1\frac{1}{3}$	-1
2	0
4	1
10	2



Domain:

$x-1>0$

$\{x \mid x>1\}$

Range: $\mathbb{R}$

Vert. Asymp: $x=1$

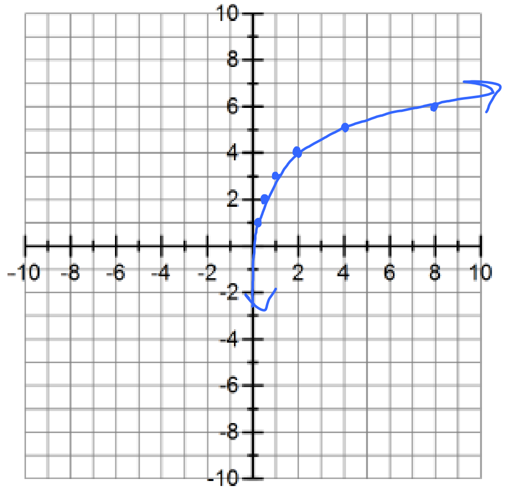
21. $f(x) = \log_2 x + 3$

$y = \log_2 x + 3$

$y - 3 = \log_2 x$

$x = 2^{y-3}$

x	y
1/4	1
1/2	2
1	3
2	4
4	5



Domain: $\{x \mid x > 0\}$

Range: $\mathbb{R}$

Vert. Asymp: $x = 0$

22. $3^{2x-5} = 27$

$3^{2x-5} = 3^3$

$2x - 5 = 3$

$2x = 8$

$\boxed{x = 4}$

23. $2 \cdot 6^{4t-1} = 72$

$6^{4t-1} = 36$

$6^{4t-1} = 6^2$

$4t - 1 = 2$

$4t = 3$

$\boxed{t = 3/4}$

24. $2e^{5x} = 6$

$e^{5x} = 3$

$\ln 3 = 5x$

$\boxed{x = \frac{\ln 3}{5} \approx 0.22}$

25. $5^{x+2} = 8^{3x-1}$

$\ln 5^{x+2} = \ln 8^{3x-1}$

$(x+2)\ln 5 = (3x-1)\ln 8$

$x \ln 5 + 2\ln 5 = 3x\ln 8 - \ln 8$

$2\ln 5 + \ln 8 = 3x\ln 8 - x\ln 5$

$2\ln 5 + \ln 8 = x(3\ln 8 - \ln 5)$

$\boxed{x = \frac{2\ln 5 + \ln 8}{3\ln 8 - \ln 5} \approx 1.145}$

26. $\log_3(x-4) = 2$

$x > 4$

$3^2 = x - 4$

$x - 4 = 9$

$\boxed{x = 13}$

27. $\log_2 x + \log_2(x-2) = 3$ $\qquad x > 0 \qquad x-2>0$

$\log_2(x(x-2)) = 3$ $\qquad x > 2$

$\log_2(x^2-2x) = 3$

$2^3 = x^2 - 2x$

$8 = x^2 - 2x$

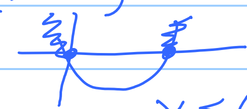
$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\boxed{x = 4}$ $\qquad \cancel{x = -2}$

28. $\log_4(x^2 - 6x) = 2$ $\qquad x^2 - 6x > 0$

$4^2 = x^2 - 6x$ $\qquad x(x-6) > 0$



$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$ $\qquad x<0$ or $x>6$

$\boxed{x=8}$ or $\boxed{x=-2}$

29. $\log(4x) = \log 3 + \log(x+5)$ $\qquad 4x > 0 \qquad x+5>0$

$\log(4x) = \log[3(x+5)]$ $\qquad x>0 \qquad x>-5$

$4x = 3(x+5)$

$4x = 3x + 15$

$\boxed{x = 15}$

30. $4^x + 2 \cdot 2^x - 8 = 0$

$(2^x)^2 + 2 \cdot 2^x - 8 = 0$

$u = 2^x$

$u^2 + 2u - 8 = 0$

$(u+4)(u-2) = 0$

$u = -4 \qquad u = 2$

$\cancel{2^x = -4} \qquad 2^x = 2$

$\boxed{x = 1}$

31. $\log_2 32 = \boxed{5}$
$2^? = 32$

32. $\log_4 \frac{1}{16} = \boxed{-2}$
$4^? = \frac{1}{16}$

33. $\ln e = \boxed{1}$
$e^? = e$

34. $\log_5 1 = \boxed{0}$
$5^? = 1$

35. $\log_7 \sqrt{7} = \boxed{\frac{1}{2}}$
$7^? = \sqrt{7}$

36. $3^{\log_3 5} = \boxed{5}$

37. $\log_6 6^{-9} = \boxed{-9}$

38. $2^5 = 32$
$\boxed{\log_2 32 = 5}$

39. $e^x = 10$
$\boxed{\ln 10 = x}$

40. $\log x = 8.3$
$\boxed{10^{8.3} = x}$

41. $\log_5 125 = x$
$\boxed{5^x = 125}$

42. $\log_8 \left(\frac{2x-3}{x^4}\right)$
$= \log_8 (2x-3) - \log_8 x^4$
$= \boxed{\log_8 (2x-3) - 4 \log_8 x}$

43. $\ln \left(\frac{x^4}{y^5 z^3}\right)$
$= \ln x^4 - \ln y^5 z^3$
$= \ln x^4 - (\ln y^5 + \ln z^3)$
$= \ln x^4 - \ln y^5 - \ln z^3$
$= \boxed{4 \ln x - 5 \ln y - 3 \ln z}$

44. $\log \left(\frac{x}{x+1}\right)^3$
$= 3 \log \left(\frac{x}{x+1}\right)$
$= 3 [\log x - \log(x+1)]$
$= \boxed{3 \log x - 3 \log(x+1)}$

45. $2 \ln x + 4 \ln(x-5) - 6 \ln(x+3)$
$= \ln x^2 + \ln (x-5)^4 - \ln (x+3)^6$
$= \ln x^2(x-5)^4 - \ln (x+3)^6$
$= \boxed{\ln \left(\frac{x^2(x-5)^4}{(x+3)^6}\right)}$

46. $3 \log_3 x - 2 \log_3 y - \log_3 z$
$= \log_3 x^3 - \log_3 y^2 - \log_3 z$
$= \log_3 \left(\frac{x^3}{y^2}\right) - \log_3 z$
$= \boxed{\log_3 \left(\frac{x^3}{y^2 z}\right)}$

47. $\log_2(x-9)$

$x-9>0$

$\boxed{\{x \mid x>9\}}$

48. $\ln(-5-x)$

$-5-x>0$

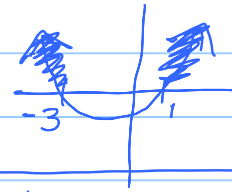
$-5>x$

$\boxed{\{x \mid x<-5\}}$

49. $\log(x^2+2x-3)$

$x^2+2x-3>0$

$(x+3)(x-1)>0$

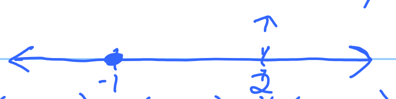


$\boxed{\{x \mid x<-3 \text{ or } x>1\}}$

50. $\log_2\left(\frac{x+1}{x-2}\right)$

$\frac{x+1}{x-2}>0$

x-int: -1 vert. asymp.: $x=2$



$(-\infty,-1)$ $(-1,2)$ $(2,\infty)$

$x=-2$ $x=0$ $x=3$

$1/4$ $-1/2$ 4

$\boxed{(-\infty,-1)\cup(2,\infty) \text{ or } \{x \mid x<-1 \text{ or } x>2\}}$

51. $\log_{13}210=\frac{\log 210}{\log 13}\approx\boxed{2.08}$

52. $\log_{1/4}28=\frac{\log 28}{\log(1/4)}\approx\boxed{-2.40}$

53. $P=\$10{,}000$ $r=0.08$ $n=365$ $t=5$

$A=P\left(1+\frac{r}{n}\right)^{nt}$ $A=10{,}000\left(1+\frac{.08}{365}\right)^{(365)(5)}=\boxed{\$14{,}917.59}$

54. $P=\$7500$ $r=0.09$ $t=8$ continuous

$A=Pe^{rt}$ $A=7{,}500e^{(0.09)(8)}=\boxed{\$15{,}408.25}$

55. $A=\$8200$ $t=3$ $r=.07$ $n=12$

$A=P\left(1+\frac{r}{n}\right)^{nt}$

$8200=P\left(1+\frac{.07}{12}\right)^{(12)(3)}$

$P=\frac{8200}{\left(1+\frac{.07}{12}\right)^{(12)(3)}}=\boxed{\$6650.85}$

56. $A = \$20{,}000$ $t=10$ $r=.05$ continuous

$A = Pe^{rt}$

$20{,}000 = Pe^{(.05)(10)}$

$P = \frac{20{,}000}{e^{(.05)(10)}} = \boxed{\$12{,}130.61}$

57. $P = \$2000$ $A = \$4000$ $r = 0.06$ $n = 4$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$4000 = 2000\left(1 + \frac{0.06}{4}\right)^{4t}$

$2 = (1.015)^{4t}$

$\log 2 = \log (1.015)^{4t}$

$\log 2 = 4t \log 1.015$

$t = \frac{\log 2}{4 \log 1.015} \approx \boxed{11.6 \text{ yrs}}$

58. $P(t) = 400e^{0.23t}$

$2000 = 400e^{0.23t}$

$5 = e^{0.23t}$

$\ln 5 = 0.23t$

$t = \frac{\ln 5}{0.23} \approx \boxed{7.0 \text{ hrs}}$

59. ½ life = 630 yrs

$.5A_0 = A_0 e^{k(630)}$

$.5 = e^{630k}$

$\ln .5 = 630k$

$k = \frac{\ln .5}{630} \approx -.0011$

$A_0 = 50$ $t = 800$

$A = 50e^{(-.0011)(800)}$

$A = \boxed{20.7 \text{ g}}$

60. ½ life = 5600 yrs

$.5A_0 = A_0 e^{k(5600)}$

$.5 = e^{5600k}$

$\ln .5 = 5600k$

$k = \frac{\ln .5}{5600} \approx -.0001238$

$A = .10A_0$

$.10A_0 = A_0 e^{-.0001238t}$

$.10 = e^{-.0001238t}$

$\ln .1 = -.0001238t$

$t = \frac{\ln .1}{-.0001238} \approx \boxed{18{,}603 \text{ yrs}}$